SHIPBUILDING

MARINE TECHNOLOGY MONTHLY

CONTENTS

VOLUME 29 - APRIL 1982 - No. 332

SHIP-MOTIONS WITH FORWARD SPEED IN INFINITE DEPTH

by

P. Guevel* and J. Bougis*

Abstract

We propose in this paper two 3D methods to solve the diffraction-radiation problem with forward speed. This physically complex problem is reduced to the determination of the potential of the flow around the immersed part of the body. Boundary conditions are linearized and calculated at the mean boundary position. The total velocity potential is obtained by summing incident, diffraction and radiation velocity potentials.

Each problem is resolved by two 3D singularities methods (mixed distribution and sources distribution). The Green function is determined with the almost perfect fluid assumption, or with the perfect fluid assumption and a radiation condition at infinity.

1. Introduction

During the last two decades, the naval industry has rapidly developed. Ships are constructed for more and more rapid transport and tankers are very large. Oceanic research and ocean exploitation are linked to the construction of floating factories and off-shore platforms.

Studies on physical models allow the prediction of seakeeping of structures, but they remain limited for two main reasons.

(i) Systematic testing in a wave tank requires a good deal of time and is expensive owing to the great number of parameters: different appendices for the same hull, cases of loads, Froude numbers, frequencies, steepness and incidences of the wave, depths of the fluid . . .

(ii) Wall effects and scale effects sometimes influence the results. Their interpretation and their extrapolation are therefore very difficult.

Thus, the joint use of physical and numerical models enables us to restrict the number of experiments and to compare different results.

Numerical models based on the strip method lead to very good results for a body at zero Froude number. Nevertheless poor results follow from its different versions for a body with forward speed. These models are limited firstly by the shape of the hulls (slender body) and secondly by the Froude number values (low speed).

Therefore, it is very important to make 3D numerical models to compute ship motions with forward speed. The first of this type of model was presented in 1977 by M.S. CHANG. It is based on the singularities method of sources distribution. We propose in this paper two 3D singularities models, the first being based on a sources and doublets distribution (mixed distribution) and the second on a sources distribution.

*) Labatoire d'Hydrodynamique navale Ecole National Superieure de Mecanique, Nantes, France. Jean Bougis, now PRINCIPIA R.D. Sophia Antipolis, France.

1) The general equation of the almost perfect fluid is

$\frac{1}{\rho}\,\overrightarrow{\text{grad}}\, p = \vec{F} - \vec{\gamma} - 2\epsilon\vec{V}$, where ϵ is a very small positive time constant.

2. Notations and hypotheses

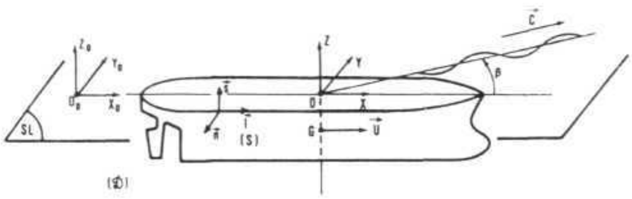
Let S be the surface of the hull, $\vec{n}$ its external normal vector, $\vec{l}$ and $\vec{s}$ its tangent vectors, where $\vec{l}$ lies in a horizontal plane and $\vec{s}$ is equal to $\vec{n} x \vec{l}$.

SL is the free surface, SF the waterplane area and C is the waterline. $\mathcal{D}$ is the whole fluid domain.

Let $(0_0\,;\, x_o, y_o, z_o)$ be a fixed frame and $(0\,;x,y,z)$ a moving system of axis bound to the mean position of the ship which translates with the uniform velocity $U.\vec{i}_x$. Oscillations are characterized by the velocity $\vec{V}$ and the angular velocity vector $\vec{\Omega}$.

Incident wave is characterized by its incidence β. its pulsation a and its amplitude a. $\vec{C}$ is its celerity.

Figure 1 shows these notations.



We shall assume the following hypotheses:

(i) The fluid is almost perfect, isovolume and its flow is irrotational[1]);

(ii) The incident wave has a small degree of steepness;

(iii) The movements of the body are small around its mean position.

3. Boundary problem

The afore mentioned hypotheses imply the existence of a velocity potential function $\phi(M;t)$ in the whole fluid domain $\mathcal{D}$. The determination of this function is reduced to the solution of a boundary problem, which can be expressed in the fixed frame, or in the moving frame.

3.1. Absolute potential expressed in the fixed frame

In the fixed frame $(O_0\,; x_o\,, y_0\,.\, z_o)$ the absolute potential is the solution of the following boundary problem

$$(1)\quad \Delta\phi(M;t) = 0 \qquad \forall M \in \mathcal{D}$$

$$(2)\quad \frac{\partial^2}{\partial t_o^2}\,\phi(M;t) + 2\epsilon\,\frac{\partial}{\partial t_o}\,\phi(M;t) + g\,\frac{\partial}{\partial z_o}\,\phi(M;t) = 0 \qquad \forall M \in SL$$

$$(3)\quad \lim_{z_o \to -\infty} \phi(M;t) = 0 \qquad \forall M \in \mathcal{D}$$

$$(4)\quad \frac{\partial}{\partial n}\,\phi(M;t) = [\vec{U} + \vec{V} + \vec{\Omega} x \vec{OM}]\,.\,\vec{n} \qquad \forall M \in S$$

$$(5)\quad \lim_{|\vec{OM}| \to \infty} [\phi(M;t) - \phi_I(M;t)] = 0 \qquad \forall M \in \mathcal{D}$$

$$(6)\quad \phi_I(M;t) = -\frac{ag}{\sigma}\,e^{k_o z_o}\,\cos[\,k_o(x_o\cos\beta + y_o\sin\beta) \quad \sigma t\,] \qquad \forall M \in \mathcal{D}$$

$$(7)\quad \sigma^2 = g k_o$$

3.2. *Absolute potential expressed in the moving frame*

In the moving frame (0 ;x, y, z) the absolute potential is the solution of the following boundary problem

$$(1')\quad \Delta\phi(M;t) = 0 \qquad \forall M \in \mathcal{D}$$

$$(2')\quad \frac{\partial^2}{\partial t^2}\,\phi(M;t) - 2U\,\frac{\partial^2}{\partial t\partial x}\,\phi(M;t) + U^2\,\frac{\partial^2}{\partial x^2}\,\phi(M;t) + 2\epsilon\,\frac{\partial}{\partial t}\,\phi(M;t)$$

$$- 2U\epsilon\,\frac{\partial}{\partial x}\,\phi(M;t) + g\,\frac{\partial}{\partial z}\,\phi(M;t) = 0 \qquad \forall M \in SL$$

$$(3')\quad \lim_{z \to -\infty} \phi(M;t) = 0 \qquad \forall M \in \mathcal{D}$$

$$(4')\quad \frac{\partial}{\partial n}\,\phi(M;t) = [\vec{U} + \vec{V} + \vec{\Omega} x \vec{OM}]\,.\,\vec{n} \qquad \forall M \in S$$

$$(5')\quad \lim_{|\vec{OM}| \to \infty} [\phi(M;t) - \phi_I(M;t)] = 0 \qquad \forall M \in \mathcal{D}$$

$$(6')\quad \phi_I(M;t) = -\frac{ag}{\sigma}\,e^{k_o z}\,\cos[\,k_o(x\cos\beta + y\sin\beta) - \omega t\,] \qquad \forall M \in \mathcal{D}$$

$$(7')\quad \omega = \sigma - k_o U\cos\beta = \sigma - \sigma^2\,\frac{U\cos\beta}{g}$$

4. Decomposition in seven simple problems

Let $\phi_p\,(M;t)$ be the velocity potential function defined by

$$(8)\quad \phi_p(M;t) = \phi(M;t) - \phi_I(M;t)$$

$\phi_p(M;t)$ must satisfy equations (1), (2), (3) and (5). The equation (9) can be substituted for (4).

$$(9)\quad \frac{\partial}{\partial n}\,\phi_p(M;t) = \vec{U}\,.\,\vec{n} - \frac{\partial}{\partial n}\,\phi_I(M;t) + [\,\vec{V} + \vec{\Omega} x \vec{OM}\,]\,.\,\vec{n}$$

The linearity hypothesis allows us to superpose different states, the solutions of which are computed separately.

The first part of the right-hand side characterizes the Neumann-Kelvin problem, the solution $\phi_W(M;t)$ of which is not dependent on the time in the relative frame. Thus it induces constant loads on the hull, the mean position of which is changed, but this problem can be treated separately.

The second part of the right-hand side of the equation (9) characterizes the diffraction problem, the solution of which is $\phi_D(M;t)$. $\phi_I(M;t)$ and $\phi_D(M;t)$ excite the ship with a sinusoidal load of encounter pulsation ω. Thus, the body performs sinusoidal oscillations of pulsation ω around its mean position.

The last part of the right-hand side of the equation (9) characterizes the radiation problem in the six modes. Solutions of these problems are linear functions of twelve constants: six amplitudes and six phases, or six components in phase with $\cos\omega t$ and six components in phase with $\sin\omega t$. We can resolve the radiation problems for only six movements of unitary speed and without phase, and we can deduce the results for six movements of unitary speed and with a phase of $\frac{\pi}{2}$.

Thus we have to resolve only seven simple problems: six for the radiation (one in each mode) and one for the diffraction.

5. Determination of the movements

After having computed the solution ϕ_D(M;t) of the diffraction problem and the six solutions ϕ_{Rj} $(M;t) j \in [1,6]$ of the radiation problems, we can compute the pressure on the hull with the Lagrange linearized formula. We obtain

$$(10)\ p = -\rho \left[\frac{\partial}{\partial t}\phi(M;t) - U\frac{\partial}{\partial x}\phi(M;t) \right]$$

where ρ is the volume mass of the fluid.

From that time onwards, we can write the Newton equation. Its terms are the following forces and moments

$$\begin{cases} F \text{ Hydrodynamic} = F \text{ Excitation} + F \text{ Radiation} \\ \mathcal{M} \text{ Hydrodynamic} = \mathcal{M} \text{ Excitation} + \mathcal{M} \text{ Radiation} \end{cases}$$

$$\begin{cases} F \text{ Hydrostatis} \\ \mathcal{M} \text{ Hydrostatic} \end{cases}$$

$$\begin{cases} F \text{ Inertia} \\ \mathcal{M} \text{ Inertia} \end{cases}$$

Here, the terms of radiation, of hydrostatic and of inertia are linear functions of the twelve unknowns.

Thus we have twelve linear equations whereas the movements are the twelve unknowns.

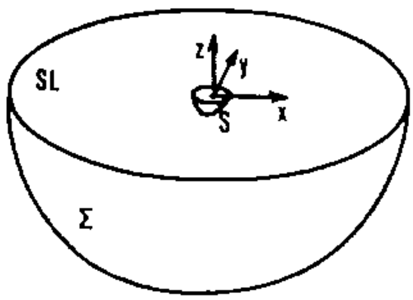


Figure 2.

6. Intergral expression of the velocity potential

Let Σ be the surface of a hemisphere defined by the Figure 2.

The application of the third Green formula in the limited domain $\mathcal{D}$ gives us

$$(11)\ -\frac{1}{4\pi}\iint_{\partial\mathcal{D}} \left[\frac{\partial}{\partial n'}\phi(M';t)\frac{1}{|MM'|} - \phi(M';t)\frac{\partial}{\partial n'}\left(\frac{1}{|MM'|}\right) \right] dS(M') = \begin{cases} \phi(M;t) & \forall M \in \mathcal{D} \\ \tfrac{1}{2}\phi(M;t) & \forall M \in \partial\mathcal{D} \\ 0 & \forall M \notin \mathcal{D} \cup \partial\mathcal{D} \end{cases}$$

When the radius of Σ tends to infinity, we have only

$$(12)\ -\frac{1}{4\pi}\iint_{SL \cup S} \left[\frac{\partial}{\partial n'}\phi(M';t)\frac{1}{|MM'|} - \phi(M';t)\frac{\partial}{\partial n'}\left(\frac{1}{|MM'|}\right) \right] dS(M') = \begin{cases} \phi(M;t) & \forall M \in \mathcal{D} \\ \tfrac{1}{2}\phi(M;t) & \forall M \in SL \cup S \\ 0 & \forall M \notin \mathcal{D} \cup SL \cup S \end{cases}$$

The same formula applied in the domain $\mathcal{D}'$ limited by *SL*, *S* and Σ' symmetric of Σ in relation to *SL* gives us

$$(13)\ \frac{1}{4\pi}\iint_{SL \cup S} \left[\frac{\partial}{\partial n'}\phi'(M';t)\frac{1}{|MM'|} - \phi'(M';t)\frac{\partial}{\partial n'}\left(\frac{1}{|MM'|}\right) \right] dS(M') = \begin{cases} 0 & \forall M \in \mathcal{D} \\ \tfrac{1}{2}\phi'(M;t) & \forall M \in SL \cup S \\ \phi'(M;t) & \forall M \in \mathcal{D}' \end{cases}$$

where $\phi(M;t)$ has the same behaviour in $\mathscr{D}'$ as $\phi(M;t)$ in $\mathscr{D}$.

The sign is changed because $\vec{n}'$ is here the internal normal vector,

The integrals on the surfaces Σ and Σ' are equal to zero as a consequence of equations (3) and (5).

After the superposition of equations (12) and (14), we obtain the general integral expression of the velocity potential function

$$(14)\quad \frac{1}{4\pi}\iint_{SL\,U\,S}\left[\frac{\partial}{\partial n'}[\phi(M';t)-\phi'(M';t)]\frac{1}{|MM'|}-[\phi(M';t)-\phi'(M';t)]\frac{\partial}{\partial n'}\left(\frac{1}{|MM'|}\right)\right]dS(M') = \begin{cases}\phi(M;t) & \forall M\in\mathscr{D}\\ \tfrac{1}{2}[\phi(M;t)+\phi'(M,t)] & \forall M\in SL\;U\;S\\ \phi'(M;t) & \forall M\in\mathscr{D}'\end{cases}$$

We shall decompose all the functions on the temporal base function $\cos\omega t$ and $\sin\omega t$, as $\phi(M;t)$

$$(15)\quad \phi(M;t)=\phi^{*}(M)\cos\omega t+\phi^{**}(M)\sin\omega t$$

Now, we can introduce the Green function. Let $g(M, M';t)$ to be a harmonic function, definite, bound and twice differentiable with respect to M and M' in the domain $\mathscr{D}$ We suppose that this function is regular at infinity. If M is a point of $\mathscr{D}'$, then we *take* $g(N, M';t)$ where N is the symmetric point of M in regard to SL We can apply the second Green formula in the domain $\mathscr{D}$ with the functions ϕ and $g(M, M';t)$ and the same formula in the domain $\mathscr{D}'$ with the functions ϕ' and $g(M, N';t)$. We shall note g_c the function relative to ϕ^* and ϕ'^*, and g_s the function relative to ϕ^{**} and ϕ'^{**}. All these functions are harmonic, then the volume integrals are equal to zero.

Taking into account the fact that the two points M and N are equal on SL, one obtains

$$\begin{aligned}(16)\quad &-\frac{1}{4\pi}\iint_{SL\,U\,S}\frac{\partial}{\partial n'}[\phi^{*}(M')-\phi'^{*}(M')]\left[\frac{\cos\omega t}{|MM'|}+g_c(M,M';t)\right]dS(M')\\ &+\frac{1}{4\pi}\iint_{SL\,U\,S}[\phi^{*}(M')-\phi'^{*}(M')]\frac{\partial}{\partial n'}\left[\frac{\cos\omega t}{|MM'|}+g_c(M,M';t)\right]dS(M')\\ &-\frac{1}{4\pi}\iint_{SL\,U\,S}\frac{\partial}{\partial n'}[\phi^{**}(M')-\phi'^{**}(M')]\left[\frac{\sin\omega t}{|MM'|}+g_s(M,M';t)\right]dS(M')\\ &+\frac{1}{4\pi}\iint_{SL\,U\,S}[\phi^{**}(M')-\phi'^{**}(M')]\frac{\partial}{\partial n'}\left[\frac{\sin\omega t}{|MM'|}+g_s(M,M';t)\right]dS(M')\\ &=\begin{cases}\phi(M;t) & \forall M\in\mathscr{D}\\ \tfrac{1}{2}[\phi(M;t)+\phi'(M;t)] & \forall M\in SL\;U\;S\\ \phi'(M;t) & \forall M\in\mathscr{D}'\end{cases}\end{aligned}$$

We should like to determine the functions g_c and g_s in order that the integral on SL is equal to zero, but this is not possible because g_c and g_s would depend on the hull. Then we must determine the Green function in order that the free surface condition is verified on the whole plane of equation z = 0.

If $\left[\frac{\cos\omega t}{|MM'|}+g_c(M,M';t)\right]$ and $\left[\frac{\sin\omega t}{|MM'|}+g_s(M,M';t)\right]$ both satisfy the equation (2), the following equivalence results

$$(17)\quad g_s(M,M',t)=-g_c^{**}(M,M')\cos\omega t+g_c^{*}(M,M')\sin\omega t$$

Thus we shall note only $g(M, M';t)$ the Green function $g_c(M, M';t)$ and we shall write

$$(18)\quad G(M,M';t)=G^{*}(M,M')\cos\omega t+G^{**}(M,M')\sin\omega t=\frac{\cos\omega t}{|MM'|}+g(M,M';t)$$

Since the free surface condition is fulfilled on the whole plane *(SL* and *SF)*, the integral on SL is not equal to zero, but we can transform it with the relation (2) connecting the different derivatives. Indeed on SL, we have the following relation

$$(19)\quad \frac{\partial}{\partial n}=-\frac{\partial}{\partial z}=\frac{1}{g}\frac{\partial^2}{\partial t^2}-\frac{2U}{g}\frac{\partial^2}{\partial t\partial x}+\frac{U^2}{g}\frac{\partial^2}{\partial x^2}$$

In the line integral on C, all the terms without spatial derivatives of $G(M, M';t)$ are equal to zero (to collate

15). After an integration by part of the other terms, one eventually obtains:

$$
\begin{aligned}
(20)\quad & -\frac{1}{4\pi}\iint_S \left\{ \frac{\partial}{\partial n'}[\phi^* - \phi'^*][G^*\cos\omega t + G^{**}\sin\omega t] + \frac{\partial}{\partial n'}[\phi^{**} - \phi'^{**}][G^*\sin\omega t - G^{**}\cos\omega t] \right\} dS(M') \\
& + \frac{1}{4\pi}\iint_S \left\{ [\phi^* - \phi'^*]\frac{\partial}{\partial n'}\{G^*\cos\omega t + G^{**}\sin\omega t\} + [\phi^{**} - \phi'^{**}]\frac{\partial}{\partial n'}\{G^*\sin\omega t - G^{**}\cos\omega t\} \right\} dS(M') \\
& - \frac{U}{2\pi g}\int_C \left\{ [\phi^* - \phi'^*]\frac{\partial}{\partial t}[G^*\cos\omega t + G^{**}\sin\omega t] + [\phi^{**} - \phi'^{**}]\frac{\partial}{\partial t}[G^*\sin\omega t - G^{**}\cos\omega t] \right\} dy' \\
& + \frac{U^2}{4\pi g}\int_C \left\{ [\phi^* - \phi'^*]\frac{\partial}{\partial x'}[G^*\cos\omega t + G^{**}\sin\omega t] + [\phi^{**} - \phi'^{**}]\frac{\partial}{\partial x'}[G^*\sin\omega t - G^{**}\cos\omega t] \right\} dy' \\
& + \frac{U^2}{4\pi g}\int_C \left\{ [\phi^* - \phi'^*]\, d\,[G^*\cos\omega t + G^{**}\sin\omega t] + [\phi^{**} - \phi'^{**}]\, d\,[G^*\sin\omega t - G^{**}\cos\omega t] \right\} (\vec{\ell}'.\vec{i}_x)(\vec{\ell}'.\vec{i}_y) \\
& - \frac{U^2}{4\pi g}\int_C \left\{ \frac{\partial}{\partial n'}[\phi^* - \phi'^*][G^*\cos\omega t + G^{**}\sin\omega t] + \frac{\partial}{\partial n'}[\phi^{**} - \phi'^{**}][G^*\sin\omega t - G^{**}\cos\omega t] \right\} (\vec{n}'.\vec{i}_x)\, dy' \\
& - \frac{U^2}{4\pi g}\int_C \left\{ \frac{\partial}{\partial s'}[\phi^* - \phi'^*][G^*\cos\omega t + G^{**}\sin\omega t] + \frac{\partial}{\partial s'}[\phi^{**} - \phi'^{**}][G^*\sin\omega t - G^{**}\cos\omega t] \right\} (\vec{s}'.\vec{i}_x)\, dy' \\
& = \begin{cases} \phi^*(M)\cos\omega t + \phi^{**}(M)\sin\omega t & \forall M \in \mathcal{D} \\ \tfrac{1}{2}[\phi^*(M) + \phi'^*(M)]\cos\omega t + \tfrac{1}{2}[\phi^{**}(M) + \phi'^{**}(M)]\sin\omega t & \forall M \in SL \cup S \\ \phi'^*(M)\cos\omega t + \phi'^{**}(M)\sin\omega t & \forall M \in \mathcal{D}' \end{cases}
\end{aligned}
$$

7. Mixed distribution (sources and doublets)

We can choose arbitrarily the function $\phi'(M;t)$ defined in $\mathcal{D}'$. Nevertheless $\phi'(M;t)$ must satisfy certain conditions in order that the aforementioned formulae are allowed.

If we take the particular following case

$$(21)\ \phi'(M;t) \equiv 0 \Leftrightarrow \phi'^*(M) \equiv 0 \text{ and } \phi'^{**}(M) \equiv 0 \qquad \forall M \in \mathcal{D}' \cup S \cup SL$$

we have, of course, a harmonic function which satisfies all the necessary conditions at infinity and the free surface condition.

Writing the velocity potential $\phi(M;t)$ in each point M of the hull, we obtain an integral equation where the unknown is $\phi(M;t)$ on S.

Thus we have the superficial singularities distribution kinematically equivalent to the hull

$$(22)\ \mu^*(M) = -\phi^*(M)\ ;\ \mu^{**}(M) = -\phi^{**}(M) \qquad M \in S$$

$$(23)\ \sigma^*(M) = \frac{\partial}{\partial n}\phi^*(M)\ ;\ \sigma^{**}(M) = \frac{\partial}{\partial n}\phi^{**}(M) \qquad M \in S$$

where the doublets distribution is unknown and the sources distribution is known. For the diffraction problem we have

$$(24)\ \frac{\partial}{\partial n}\phi_D^*(M) = -\frac{\partial}{\partial n}\phi_I^*(M)\ ;\ \frac{\partial}{\partial n}\phi_D^{**}(M) = -\frac{\partial}{\partial n}\phi_I^{**}(M) \qquad M \in S$$

and for the radiation problem we have

$$(25)\ \frac{\partial}{\partial n}\phi_{R_j}^*(M) = \vec{V}_{E_j}^* . \vec{n}\ ;\ \frac{\partial}{\partial n}\phi_{R_j}^{**}(M) = \vec{V}_{E_j}^{**} . \vec{n} = 0 \qquad M \in S$$

These distributions satisfy the following equations

$$
\begin{aligned}
(26)\quad & \frac{1}{2}\mu^*(M) - \frac{1}{4\pi}\iint_S \left[\mu^*(M')\frac{\partial}{\partial n'}G^*(M,M') - \mu^{**}(M')\frac{\partial}{\partial n'}G^{**}(M,M') \right] dS(M') \\
& + \frac{U\omega}{2\pi g}\int_C [\mu^*(M')G^*(M,M') + \mu^{**}(M')G^{**}(M,M')]\, dy' \\
& - \frac{U^2}{4\pi g}\int_C \left[\mu^*(M')\frac{\partial}{\partial x'}G^*(M,M') - \mu^{**}(M')\frac{\partial}{\partial x'}G^{**}(M,M') \right] dy' \\
& \frac{U^2}{4\pi g}\int_C [\mu^*(M')\, dG^*(M,M') - \mu^{**}(M')\, dG^{**}(M,M')]\ (\vec{\ell}'.\vec{i}_x)(\vec{\ell}'.\vec{i}_y)
\end{aligned}
$$

$$+\frac{U^2}{4\pi g}\int_C\left[\frac{\partial\mu^*}{\partial s'}(M')\,G^*(M,M')-\frac{\partial\mu^{**}}{\partial s'}(M')\,G^{**}(M,M')\right](\vec{s}'\,.\,\vec{i}_x)\,dy'$$

$$=\frac{1}{4\pi}\iint_S[\sigma^*(M')\,G^*(M,M')-\sigma^{**}(M')\,G^{**}(M,M')]\,dS(M')$$

$$+\frac{U^2}{4\pi g}\int_C[\sigma^*(M')\,G^*(M,M')-\sigma^{**}(M')\,G^{**}(M,M')]\,(\vec{n}'\,.\,\vec{i}_x)\,dy'$$

$$(27)\ \frac{1}{2}\mu^{**}(M)-\frac{1}{4\pi}\iint_S\left[\mu^{**}(M')\frac{\partial}{\partial n'}G^*(M,M')+\mu^*(M')\frac{\partial}{\partial n'}G^{**}(M,M')\right]dS(M')$$

$$+\frac{U\omega}{2\pi g}\int_C[\mu^{**}(M')\,G^{**}(M,M')-\mu^*(M')\,G^*(M,M')]\,dy'$$

$$-\frac{U^2}{4\pi g}\int_C\left[\mu^{**}(M')\frac{\partial}{\partial x'}G^*(M,M')+\mu^*(M')\frac{\partial}{\partial x'}G^{**}(M,M')\right]dy'$$

$$-\frac{U^2}{4\pi g}\int_C[\mu^{**}(M')\,dG^*(M,M')+\mu^*(M')\,dG^{**}(M,M')]\,(\vec{\tau}'\,.\,\vec{i}_x)(\vec{\tau}'\,\vec{i}_y)$$

$$+\frac{U^2}{4\pi g}\int_C\left[\frac{\partial\mu^{**}}{\partial s'}(M')\,G^*(M,M')+\frac{\partial\mu^*}{\partial s'}(M')\,G^{**}(M,M')\right](\vec{s}'\,.\,\vec{i}_x)\,dy'$$

$$=\frac{1}{4\pi}\iint_S[\sigma^{**}(M')\,G^*(M,M')+\sigma^*(M')\,G^{**}(M,M')]\,dS(M')$$

$$+\frac{U^2}{4\pi g}\int_C[\sigma^{**}(M')\,G^*(M,M')+\sigma^*(M')\,G^{**}(M,M')]\,(\vec{n}'\,.\,\vec{i}_x)\,dy'$$

When the hull cuts vertically the free surface, the term $(\vec{s}'\,.\,\vec{i}_x)$ cancels and the integrals on C with the surface gradient of $\mu^*(M)$ and $\mu^{**}(M)$ vanish.

8. Sources distribution

If we consider the other following particular case

$$(28)\ \phi'(M;t)\equiv\phi(M;t)\Leftrightarrow\phi'^*(M)\equiv\phi^*(M)\ \text{and}\ \phi'^{**}(M)\equiv\phi^{**}(M)\qquad\forall M\in S\cup SL$$

$\phi'(M;t)$ is a harmonic function and satisfies, as $\phi(M;t)$ the free surface condition and the conditions at infinity.

Writing the velocity $\frac{\partial}{\partial n}\phi(M;t)$ in each point M of the hull, we obtain an integral equation where the unknown is the discontinuity of the normal derivative of the potential of the hull.

Thus the superficial singularities distribution kinematically equivalent to the hull is

$$(29)\ \sigma^*(M)=\frac{\partial}{\partial n}\phi^*(M)-\frac{\partial}{\partial n}\phi'^*(M)\ ;\ \sigma^{**}(M)=\frac{\partial}{\partial n}\phi^{**}(M)-\frac{\partial}{\partial n}\phi'^{**}(M)\qquad M\in S$$

This distribution of singularities satisfies (30) and (31).

$$(30)\ \frac{1}{2}\sigma^*(M)-\frac{1}{4\pi}\iint_S\left[\sigma^*(M')\frac{\partial}{\partial n}G^*(M,M')-\sigma^{**}(M')\frac{\partial}{\partial n}G^*(M,M')\right]dS(M')$$

$$-\frac{U^2}{4\pi g}\int_C\left[\sigma^*(M')\frac{\partial}{\partial n}G^*(M,M')-\sigma^{**}(M')\frac{\partial}{\partial n}G^{**}(M,M')\right](\vec{n}'\,.\,\vec{i}_x)\,dy'=\frac{\partial}{\partial n}\phi^*(M)$$

$$(31)\ \frac{1}{2}\sigma^{**}(M)-\frac{1}{4\pi}\iint_S\left[\sigma^{**}(M')\frac{\partial}{\partial n}G^*(M,M')+\sigma^*(M')\frac{\partial}{\partial n}G^{**}(M,M')\right]dS(M')$$

$$-\frac{U^2}{4\pi g}\int_C\left[\sigma^{**}(M')\frac{\partial}{\partial n}G^*(M,M')+\sigma^*(M')\frac{\partial}{\partial n}G^{**}(M,M')\right](\vec{n}'\,.\,\vec{i}_x)\,dy'=\frac{\partial}{\partial n}\phi^{**}(M)$$

The right-hand sides of these equations are given by the formulae (24) and (25).

After the resolution of these equations, the velocity potential in each point of the space can be computed with the help of the following integral expressions

$$(32)\ -\frac{1}{4\pi}\iint_S[\sigma^*(M')\,G^*(M,M')-\sigma^{**}(M')\,G^{**}(M,M')]\,dS(M')$$

$$-\frac{U^2}{4\pi g}\int_C[\sigma^*(M')\,G^*(M,M')-\sigma^{**}(M')\,G^{**}(M,M')]\,(\vec{n}'\,.\,\vec{i}_x)\,dy'$$

$$= \begin{cases} \phi^*(M) & \forall M \in \mathcal{D} \\ \tfrac{1}{2}\phi^*(M) + \tfrac{1}{2}\phi'^*(M) = \phi^*(M) & \forall M \in SL \cup S \\ \phi'^*(M) & \forall M \in \mathcal{D}' \end{cases}$$

$$(33)\quad -\frac{1}{4\pi}\iint_S [\sigma^{**}(M')\,G^*(M,M') + \sigma^*(M')\,G^{**}(M,M')]\,dS(M')$$

$$-\frac{U^2}{4\pi g}\int_C [\sigma^{**}(M')\,G^*(M,M') + \sigma^*(M')\,G^{**}(M,M')]\,(\vec{n}'\cdot\vec{i}_x)\,dy'$$

$$= \begin{cases} \phi^{**}(M) & \forall M \in \mathcal{D} \\ \tfrac{1}{2}\phi^{**}(M) + \tfrac{1}{2}\phi'^{**}(M) = \phi^*(M) & \forall M \in SL \cup S \\ \phi'^{**}(M) & \forall M \in \mathcal{D}' \end{cases}$$

9. Green function

In the past, the determination of the Green function relative to the diffraction-radiation problem with forward speed has interested numerous writers. Thus M.D. Haskind (1946), R. Brard (1948), then T. Hanaoka (1953), L.N. Sretenskii (1954) and T.H. Havelock (1958) have given different formulations of this function. J.V Wehausen and E.V. Laitone (1960) have analysed all these formulations,

We prefer to use the Green function formulation proposed by P. Guevel, J. Bougis and D.C. Hong (1979) which is adapted to the demands of numerical treatment and asymptotic expansion.

We propose here the Green function built in the most general case for an infinite depth. To construct this function, we make the presumption that $G(M, M'; t)$ can be represented by the following integral expression,

$$(34)\quad G(M,M';t) = \frac{f(t)}{|MM'|} + \lim_{\epsilon\to o^+}\frac{1}{\pi} Re\left\{\int_{-\pi/2}^{+\pi/2} d\theta \int_o^\infty \hat{g}(\theta,k,\epsilon;t)\,e^{kz}\,e^{ik(x\cos\theta + y\sin\theta)}\,k\,dk\right\}$$

where $f(t)$ is any function of the time as the coordinates $x'_0(t)$, $y'_0(t)$, $z'_0(t)$ of M'. This choice is justified by the following three reasons.

(i) This function is harmonic in the whole domain $\mathcal{D}$ -M' as $\frac{1}{|MM'|}$, since $e^{kz_o}\,e^{ik(x\cos\theta + y\sin\theta)}$ is the elementary solution of Laplace equation in the domain $\mathcal{D}$ definde by $z_o < 0$, obtained by variables separation procedure:

(ii) This function verifies $\lim_{z_o\to-\infty} G(M, M'; t) = 0$ in the whole domain $\mathcal{D}$;

(iii) This function must verify the condition $\lim_{|MM'|\to\infty} G(M, M'; t) = 0$ in the whole domain $\mathcal{D}$ if we do not want to write divergent integrals. Nevertheless we shall have to check later that the solution found conforms to this,

In these conditions, the function $\hat{g}(0,k,e;t)$ will be determined by imposing that $G(M,M';t)$ fulfills the linearized free surface equation. The general solution is, for a build in static equilibrium at time t.

$$(35)\quad G(M,M';t) = \left[\frac{1}{|MM'|} - \frac{1}{|MN'|}\right] + \lim_{\epsilon\to o^+}\frac{1}{\pi} Re\left\{\int_{-\pi/2}^{+\pi/2} d\theta \int_o^\infty \hat{h}(\theta,k,\epsilon;t)\,e^{kz}\,e^{ik(x\cos\theta + y\sin\theta)}k\,dk\right\}$$

N' denoted the symmetrical point of M' in relation to the free surface and

$$(36)\quad \hat{h}(\theta,k,\epsilon;t) = -\int_{t_1}^{t} i\sqrt{\frac{g}{k}}\,f(\tau)\,e^{k[z'(\tau) - i(x'(\tau)\cos\theta + y'(\tau)\sin\theta)]}\,[e^{\lambda_1(t-\tau)} - e^{\lambda_2(t-\tau)}]\,d\tau$$

with

$$(37)\quad \lambda_1 = -\epsilon + i\sqrt{gk}\ ;\ \lambda_2 = -\epsilon - i\sqrt{gk}$$

In the particular case where $f(t) = \cos\omega t$, $x'_0(t) = x'(t_1) + Ut$, $y'_0(t) = y'_0(t_1)$, $z'_o(t) = z'_0(t)$ and t_1 goes to $-\infty$, we obtain

(38) $G(M, M'; t) = G_0(M, M'; t) + G_1(M, M'; t) + G_2(M, M'; t)$

with

$$(39)\ G_o(M, M'; t) = \left[\frac{1}{|MM'|} - \frac{1}{|MN'|} \right] \cos\omega t$$

$$(40)\ G_1(M, M'; t) = \lim_{\tilde{\epsilon} \to o^+} \left[\frac{1}{\pi \ell} Re \left\{ e^{i\omega t} \int_{-\pi/2}^{+\pi/2} d\theta \int_o^\infty \frac{e^{K|Z+Z'+i\Omega|} K\, dK}{(\tilde{\omega} - FK\cos\theta)^2 - K - 2i\tilde{\epsilon}(\tilde{\omega} - FK\cos\theta)} \right\} \right]$$

$$(41)\ G_2(M, M'; t) = \lim_{\tilde{\epsilon} \to o^+} \left[\frac{1}{\pi \ell} Re \left\{ e^{-i\omega t} \int_{-\pi/2}^{+\pi/2} d\theta \int_o^\infty \frac{e^{K|Z+Z'+i\Omega|} K\, dK}{(\tilde{\omega} + FK\cos\theta)^2 - K + 2i\tilde{\epsilon}(\tilde{\omega} + FK\cos\theta)} \right\} \right]$$

In this expression the characteristic sizes of flow are adimensionalised in relation to a reference length ℓ which is, for example, the ship waterline length

$$(42)\ X = \frac{x}{\ell} \quad ; \quad Y = \frac{y}{\ell} \quad ; \quad Z = \frac{z}{\ell} \quad ; \quad \nu = \tilde{\omega} F$$

$$K = k\ell \quad ; \quad \tilde{\omega} = \omega \sqrt{\frac{\ell}{g}} \quad ; \quad \tilde{\epsilon} = \epsilon \sqrt{\frac{\ell}{g}} \quad ; \quad F = \frac{U}{\sqrt{g\ell}}$$

$$\Omega = (X - X')\cos\theta + (Y - Y')\sin\theta \quad ; \quad \Omega' = (X - X')\cos\theta - (Y - Y')\sin\theta$$

Let K_1 and K_2 be the poles of the integrand of $G_1(M, M'; t)$ and K_3 and K_4 those of the integrand of $G_2(M,M';t)$, Then the analysis of the denominators gives us

$$(43)\ K_1 = \frac{1 + 2\nu\cos\theta - \sqrt{1 + 4\nu\cos\theta}}{2F^2\cos^2\theta} - i\tilde{\epsilon}_1 \qquad \text{with } \tilde{\epsilon}_1 > 0$$

$$(44)\ K_2 = \frac{1 + 2\nu\cos\theta + \sqrt{1 + 4\nu\cos\theta}}{2F^2\cos^2\theta} - i\tilde{\epsilon}_2 \qquad \text{with } \tilde{\epsilon}_2 > 0$$

$$(45)\ \begin{cases} K_3 = \dfrac{1 - 2\nu\cos\theta - \sqrt{1 - 4\nu\cos\theta}}{2F^2\cos^2\theta} + i\tilde{\epsilon}_3 & \text{with } \tilde{\epsilon}_3 > 0 \text{ if } \nu < 1/4 \\ K_3 = \dfrac{1 - 2\nu\cos\theta - i\sqrt{4\nu\cos\theta - 1}}{2F^2\cos^2\theta} & \text{if } \nu > 1/4 \end{cases}$$

$$(46)\ \begin{cases} K_4 = \dfrac{1 - 2\nu\cos\theta + \sqrt{1 - 4\nu\cos\theta}}{2F^2\cos^2\theta} - i\tilde{\epsilon}_4 & \text{with } \epsilon_4 > 0 \text{ if } \nu < 1/4 \\ K_4 = \dfrac{1 - 2\nu\cos\theta + i\sqrt{4\nu\cos\theta - 1}}{2F^2\cos^2\theta} & \text{if } \nu > 1/4 \end{cases}$$

Integrate with respect to K, and let $\tilde{\epsilon}$ tend to zero, and the following expression results

$$(47)\ G_1(M, M'; t) = \frac{1}{\pi\ell} Re \left\{ e^{i\omega t} \int_o^{\pi/2} \frac{K_1[G1(K_1\xi) + G1(K_1\xi')] - K_2[(G1(K_2\xi) + G1(K_2\xi')]}{\sqrt{1 + 4\nu\cos\theta}} d\theta \right\}$$

$$(48)\ G_2(M, M'; t) = -\frac{1}{\pi\ell} Re \left\{ e^{-i\omega t} \left[\int_o^{\theta_c} i \frac{K_3[G2(K_3\xi) + G2(K_3\xi')] - K_4[G2(K_4\xi) + G2(K_4\xi')]}{\sqrt{4\nu\cos\theta - 1}} d\theta \right. \right.$$
$$+ \int_{\theta'_c}^{\theta_c} i \frac{K_3[G1(K_3\xi) + G1(K_3\xi')] - K_4[G3(K_4\xi) + G3(K_4\xi')]}{\sqrt{4\nu\cos\theta - 1}} d\theta$$
$$\left. \left. - \int_{\theta_c}^{+\pi/2} \frac{K_3[G3(K_3\xi) + G3(K_3\xi')] - K_4[G1(K_4\xi) + G1(K_4\xi')]}{\sqrt{1 - 4\nu\cos\theta}} d\theta \right] \right\}$$

In (48) $\cos\theta_c$ is equal to $\frac{1}{4\nu}$ and $\cos\theta'_c$ is equal to $\frac{1}{2\nu}$. The variable ξ denotes $Z + Z' + i\Omega$ and ξ' denotes $Z + Z' + i\Omega'$. $G1$, $G2$ and $G3$ represent the following modified exponential integral functions

$$(49)\ G1(\xi) = e^{\xi} E_1(\xi) \qquad 0 < Arg(\xi) < 2\pi$$

$$(50)\ G2(\xi) = e^{\xi} E_1(\xi) \qquad -\pi < Arg(\xi) < +\pi$$

$$(51)\ G3(\xi) = e^{\xi}[E_1(\xi) + 2i\pi] \qquad 0 < Arg(\xi) < 2\pi$$

with

(52) $E_1(\xi) = E_1(\xi)$ $\qquad Im(\xi) > 0$

(53) $E_1(\xi) = E_1(\xi + i\epsilon)$ $\qquad Im(\xi) = 0$

(54) $E_1(\xi) = E_1(\xi) - 2i\pi$ $\qquad Im(\xi) < 0$

It must be emphasized that

(i) If $v \in]¼, ½]$, the first integral of the formula (48) does not. exist, and θ'_c must be replaced by zero in the second integral;

(ii) If $v = ¼$, the integral of $G_2(M, M'; t)$ is divergent;

(iii) If $v \in [0, ¼[$, the first two integrals of the formula (48) do not exist, and $\theta_{c'}$ must be replaced by zero in the third integral,

(iv) The same results would be obtained without $\tilde{\epsilon}$, but with a radiation condition at infinity, which is (5). "In the relative frame, the energy of all the wave systems move away from the body generating them".

10. Resolution of the integral equations

The integral equations will now be solved with a method of discretization. Let the hull be discretized with N panels. We shall assume that on each panel the densities of singularities a or µ are constant. Each integral on S is decomposed in a sum of N integrals which concern only the Green function or its derivates,

Thus we obtain, for example, for the sources distribution

$$(55)\ \frac{1}{2}\sigma_i^* + \sum_{j=1}^{N}[\sigma_j^* V_{ij}^* - \sigma_j^{**} V_{ij}^{**}] + \sum_{k=1}^{m}[\sigma_k^* Vn_{ik}^* - \sigma_k^{**} Vn_{ij}^{**}] = \frac{\partial}{\partial n}\phi^*(M_i)$$

$$(56)\ \frac{1}{2}\sigma_i^{**} + \sum_{j=1}^{N}[\sigma_j^{**} V_{ij}^* + \sigma_j^* V_{ij}^{**}] + \sum_{k=1}^{m}[\sigma_k^{**} Vn_{ik}^* + \sigma_k^* Vn_{ik}^{**}] = \frac{\partial}{\partial n}\phi^{**}(M_i)$$

where m is the number of the panels cutting the free surface, i the index of the panel on which the body condition is written, and

$$(57)\ V_{ij}^* = -\frac{1}{4\pi}\iint_{S_j}\frac{\partial}{\partial n_i}G^*(M_i, M_j)\,ds(M_j)\ ;\ V_{ij}^{**} = -\frac{1}{4\pi}\iint_{S_j}\frac{\partial}{\partial n_i}G^{**}(M_i, M_j)\,ds(M_j)$$

$$(58)\ Vn_{ik}^* = -\frac{U^2}{4\pi g}(\vec{n}_k \cdot \vec{i}_x)\int_{c_k}\frac{\partial}{\partial n_i}G^*(M_i, M_k)\,dy_k\ ;\ Vn_{ik}^{**} = -\frac{U^2}{4\pi g}(\vec{n}_k \cdot \vec{i}_x)\int_{c_k}\frac{\partial}{\partial n_i}G^{**}(M_i, M_k)\,dy_k$$

with M_i the control point of the panel i (for example the center of gravity of the panel i.).

We have of course N equations (55) and N equations (56). Thus we must solve a system of $2N$ equations with 27V unknowns.

For the mixed distribution, the procedure is analogous, but there are many more terms. However, we must remark that there are $2m$ supplementary unknowns, $\frac{\partial \mu_k^*}{\partial s_k}$ and $\frac{\partial \mu_k^{**}}{\partial s_k}$ when the hull cuts the free surface with an angle different from $\pi/2$.

Two methods might be envisaged to extend this second distribution of singularities in the general case.

(i) To consider these $2m$ supplementary unknowns and $2m$ supplementray equations, writing the body condition in m other points or writing that the potential is equal to zero inside the body. But these points cannot be on the waterplane because the integrals of the Green function relative to the coefficients of $\frac{\partial \mu_k}{\partial s_k}$ would be very difficult to compute,

(ii) To solve the system without these supplementary terms and procede by an iterative method. The derivatives of μ_k^* and μ_k^{**} in relation to s_k can be computed with one of the following two methods. The first is the use of the formula (20) without these terms, and the second is the finite difference between the last two panels of the hull before the free surface.

In all the cases, we have one linear system with the same number as equations that of unknowns even if we must, in the second case, repeat the resolution.

All the coefficients of influence (as (57) and (58)) are given by an integral of $G^*(M_i, M_j)$, $G^{**}(M_i, M_j)$ or its derivates in relation to n_i, n_j or x_j. Since the Green function depends on the spatial coordinates through the variable ξ(or ξ') which is a linear function of those, we can integrate analytically the Green function and its derivatives on the panels. Thus we must only integrate numerically with respect to θ.

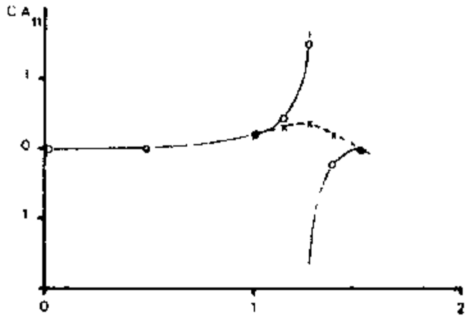


Figure 3.

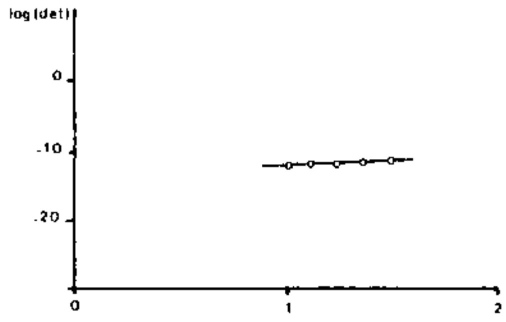


Figure 4.

11. Irregular frequencies

The first numerical tests have shown that the resolution of the system relative to the mixed distribution has irregular frequencies. These irregularities come of the inside adjoint problem; we can suppress them with two supplementary equations which arise from the fact that the potential is written equal to zero inside the body. The Figure 3 shows the differences between the results obtained without and with these equation about the surge hydrodynamic coefficients for a barge (20 panels).

For the sources distribution, no irregular frequencies have been detected, and the determinant of the system never vanished in the intervals studied. However, if this second method has irregular frequencies, they are not the same as for the first program because the two integral equations are not adjoint, as opposed to the problem at zero Froude number. Figure 4 shows the values of the determinant of the sources distribution in the interval of the irregular frequencies of the mixed distribution (20 panels).

12. Integral on Σ

We have determined $\phi(M;\ t)$ with a heuristic method, assuming that the contribution of the integral on Σ is equal to zero. Now, we must prove it. We shall give here only a sketch of the proof.

$\phi(M;\ t)$ is defined by the relation (59)

$$(59)\ \phi(M;t) = I_S[\phi,\ G]\ (M;t) \qquad M \in \mathcal{D}$$

expression in which I_s denotes the integral on the hull S and the integral on the waterline C. Let $\psi(M;\ t)$ be the function defined by the following relation

$$(60)\ \psi(M;t) = I_S[\psi,\ G]\ (M;t) + I_\Sigma[\psi,\ G]\ (M;t) \qquad M \in \mathcal{D}$$

$\phi(M;\ t)$ and $\psi(M;\ t)$ will be equal for each point M of the domain $\mathcal{D}$ if

$$(61)\ I_\Sigma[\phi,\ G]\ (M;t) \equiv o \qquad \forall M \in \mathcal{D}$$

I_Σ denoting, of course, the integral over the surface Σ including the line integral along the intersection of Σ and SL.

Since (60) results from the Green formula, it is true for any surface Σ. For the same reason $\phi(M;t)$ and $I_S[\phi,\ G]\ (M;t)$ do not depend on Σ. Thus $I_\Sigma[\phi,\ G]\ (M;t)$ is not dependent on Σ when $M \in \mathcal{D}$.

An asymptotic analysis of $\phi(M;t)$ (to collate 5) shows that if R is great enough, then

$$(62)\ \phi(M';t) = \sum_i Re \left| A'_i(S,\beta',z')\, e^{i[B'_i(\beta')R' + C'_i + D'_i\omega t]} \right| R'^{\,-n_i(\beta)} \left[1 + O\left(R'^{\,-n_i(\beta)}\right)\right]$$

$$(63)\ G(M,M';t) = \sum_j Re \left| A_j(M',\beta,z)\, e^{i[B_j(\beta)R + C_j + D_j\omega t]} \right| R^{-n_j(\beta)} \left[1 + O\left(R^{-n_j(\beta)}\right)\right]$$

where $n_i(\beta)$ denotes one of the values $\left\{\frac{1}{3}, \frac{1}{2}, 1\right\}$, C_i denotes one of the values $\left\{-\frac{\pi}{4}, 0, +\frac{\pi}{4}\right\}$ and where D_i is equal to $\pm$ 1.

Let Σ be the surface of a hemisphere defined by a great radius R'; with the notation of Figure 5, we have the relation (64) between R, R' and d.

$$(64)\ R = R'\cos(\beta' - \beta) - d\cos(\alpha - \beta')$$

Then the kerned of $I_\Sigma[\phi,\ G]\ (M;t)$ can be written as

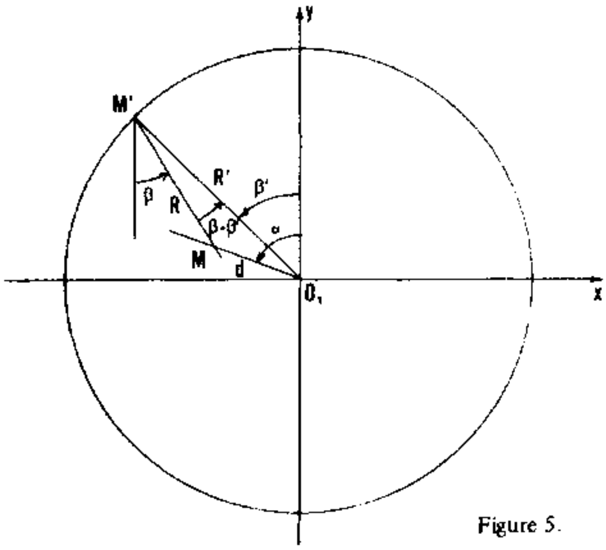


Figure 5.

$$(65)\ \sum_{ij} Re\left| A_i'(S,\beta',z')\, e^{i[B_i'(\beta')R'+C_i'+D_i'\omega t]}\right| Re\left| A_j(M',\beta,z)\, e^{i[B_j(\beta)R+C_j+D_j\omega t]}\right|$$

$$\times R'^{-(n_i(\beta')+n_j(\beta))}\ [\cos(\beta'-\beta)]^{-n_j(\beta)}\left[1+O\left(R'^{-(n_i(\beta')+n_j(\beta))}\right)\right]$$

We must note that $n_i(\beta)$ is equal to $\frac{1}{3}$ for only one value β_c of β, and the behaviour of A_i is then

$$(66)\ \frac{\sqrt{2\pi}}{\sqrt{[K_i''(\theta_i)-K_i(\theta_i)]\sin(\theta_i-\beta')+2K_i'(\theta_i)\cos(\theta_i-\beta')}}$$

then, the integration with respect to β' converges and $I_\Sigma\,[\phi, G]\ (M;t)$ can be overvalued

$$(67)\ I_\Sigma\,[\theta, G]\ (M;t) \leqslant \iint_\Sigma \sum_{ij} |Re\{A_i'(S,\beta',z')\}|\ |Re\{A_i(M',\beta,z)\}|$$

$$[\cos(\beta'-\beta)]^{-n_j(\beta)}\, R'^{\,1-(n_i(\beta')+n_j(\beta))} \times \left[1+O\left(R'^{\,1-(n_i(\beta')+n_j(\beta))}\right)\right] d\Sigma$$

If $n_i(\beta') + n_j(\beta)$ is not equal to 1, the overvaluation goes to zero when R' goes to infinity and $I_\Sigma\,[\theta, G]\ (M;t)$ is equal to zero when Σ is at infinity, thus I_Σ is always equal to zero.

If $n_i(\beta') + n_j(\beta)$ is equal to 1, I_Σ can be written as

$$(68)\ I_\Sigma(\phi, G)(M;t) = \iint_\Sigma \sum_{ij} Re\left| A_i'(S,\beta',z')\, e^{i[B_i'(\beta)R'+C_i'+D_i'\omega t]}\right|$$

$$\times Re\left| A_i(M',\beta,z)\, e^{i[B_j(\beta)R+C_j+D_j\omega t]}\right|\ [\cos(\beta'-\beta)]^{-n_j(\beta)}\, d\beta dh$$

thus $I_\Sigma\,[\phi, G]$ is a linear superposition of sinusoidal functions of R' and is independent of R' only if it is equal to zero.

13. Numerical results

We present some numerical results about two different hulls.

Firstly we have tested our two computer programs with a D.N.V. barge (rectangular parallelepiped 90m x 90m x 40m) for a Froude number equal to 0.15.

Results concerning added-mass coefficients (CM_{ij}) and damping coefficients (CA_{ij}) are given on Figures 6 and 7. They show a good agreement between the two methods with only 10 panels on the half hull The greatest differences are obtained for surge motion.

Secondly we have tested the computer program based on sources distribution with a hull series 60 C_B 0.70 for a Froude number equal to 0.20.

Results concerning added-mass coefficients (CM_{ij}) and damping coefficients (CA_{ij}) are given on Figures 8, 9 and 10 with experimental results of J.H. Vugts and numerical results of M.S. Chang. They show a good accuracy of our results with experimental measurements except for $C4_{55}$ and CM_{26} , for only 27 panels on the half hull

These last results are obtained with a correction on the normal vector which is written as $\vec{n} = \vec{n}_o + \vec{\theta} \times \vec{n}_o$.

Barge du D.N.V. (10 facettes sur la demi-carène). Profondeur infinie.

Nombre de Froude : F = 0.15

Longueur : L = 90.00 m

● Distribution mixte

o Distribution de sources

$$CM_{ii} = \frac{MA_{ii}}{\rho \nabla} \quad i \in 1,3$$

$$CM_{ii} = \frac{MA_{ii}}{\rho \nabla L^2} \quad i \in 4,6$$

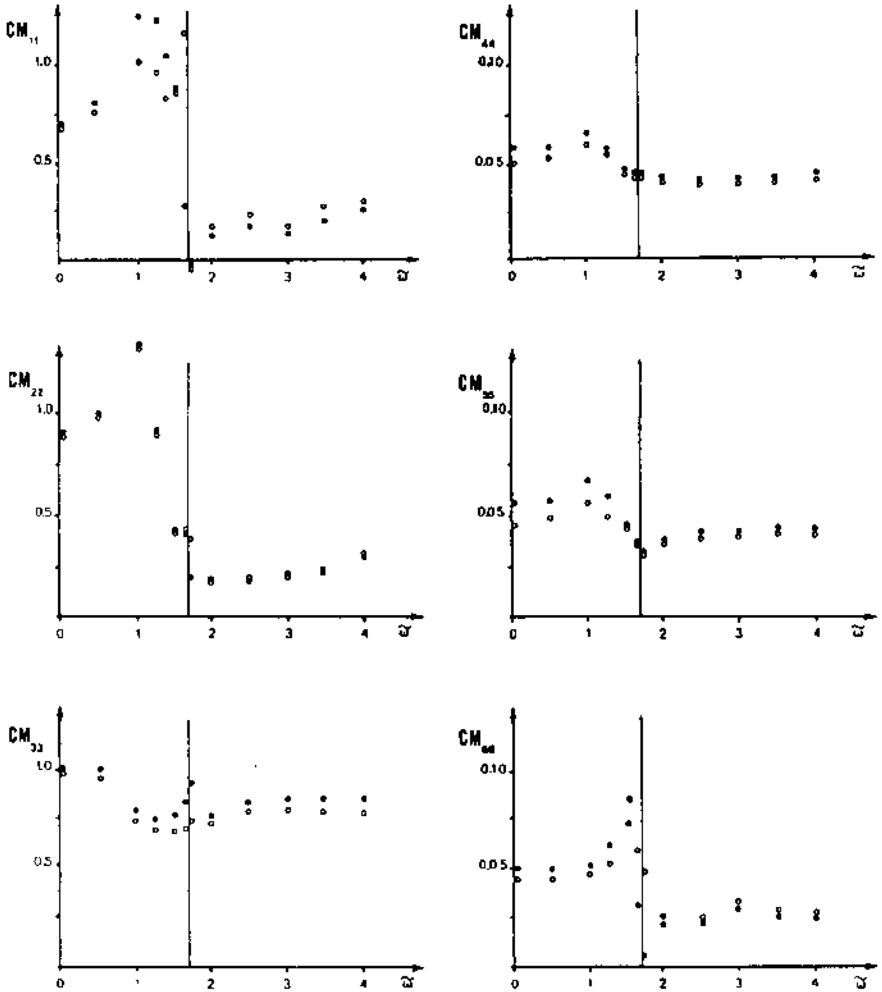


Figure 6.

Barge du D.N.V. (10 facettes sur la demi-carène). Profondeur infinie.

Nombre de Froude : F = 0.15

Longueur : L = 90.00 m

● Distribution mixte

o Distribution de sources

$$CA_{ii} = \frac{TA_{ii}}{\rho \nabla \omega} \quad i \in 1,3$$

$$CA_{ii} = \frac{TA_{ii}}{\rho \nabla L^2 \omega} \quad i \in 4,6$$

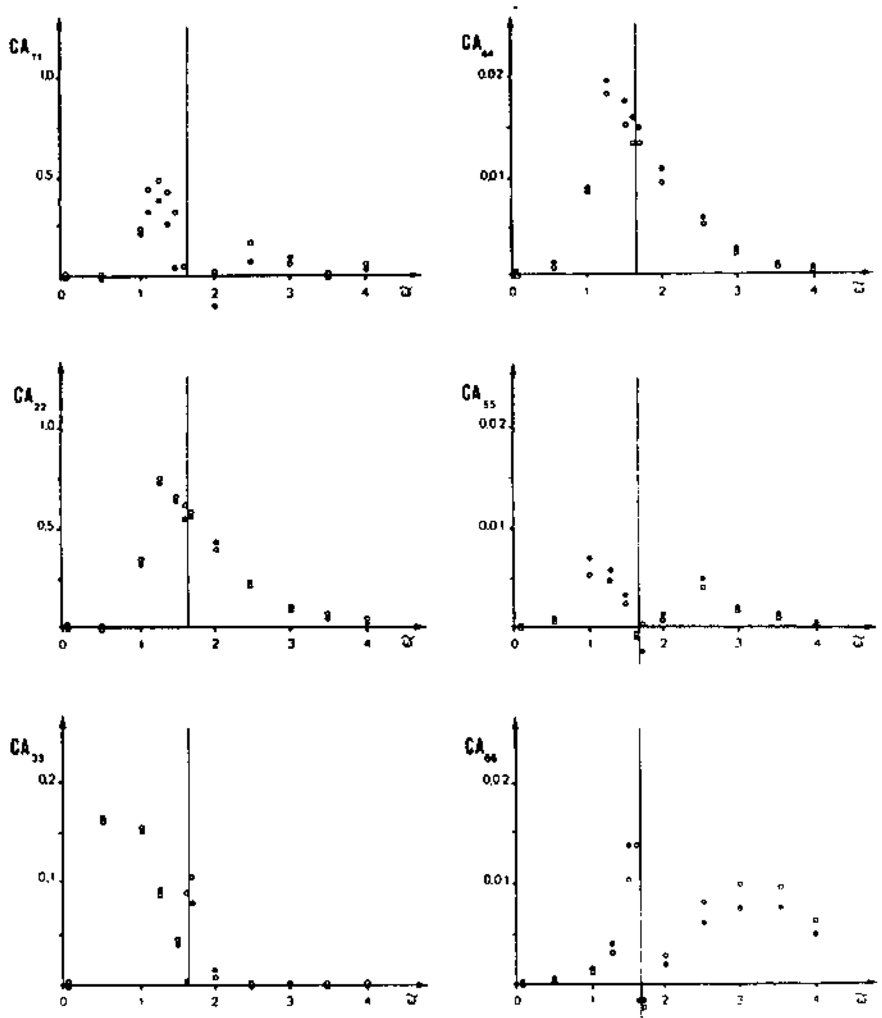


Figure 7.

Séries 60 C_B 0.70 (27 facettes sur la demi-carène). Profondeur infinie.

Nombre de Froude : F = 0.20

Longueur : L = 121.92 m

$CM_{ii} = \frac{MA_{ii}}{\rho\nabla}$ i ∈ 1,3

$CM_{ii} = \frac{MA_{ii}}{\rho\nabla L^2}$ i ∈ 4,6

- • résultats de J.H.VUGTS (modèle complet)
- o résultats de J.H.VUGTS (somme des sections)
- c résultats de M.S.CHANG
- x résultats des présents calculs
- —— Méthode des tranches

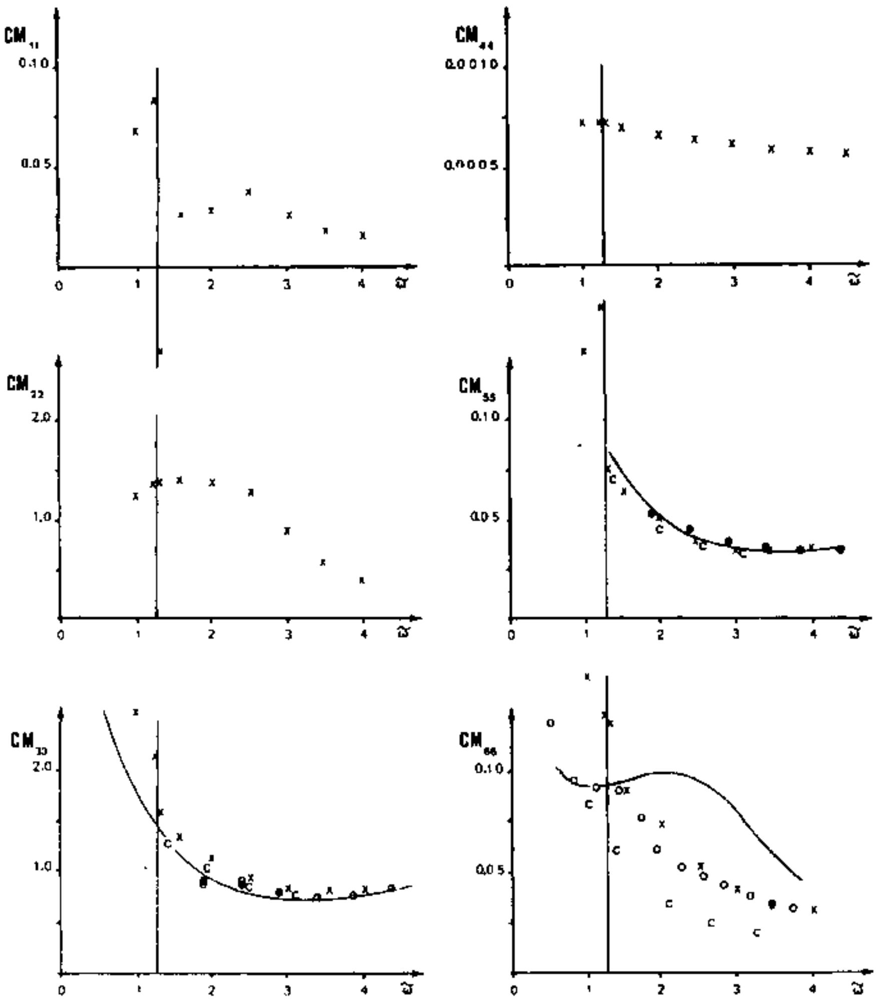


Figure 8.

Séries 60 C_B 0.70 (27 facettes sur la demi-carène). Profondeur infinie.

Nombre de Froude : F = 0.20

Longueur : L = 121.92 m

$CA_{ii} = \frac{TA_{ii}}{\rho\nabla\sqrt{g/L}}$ i ∈ 1,3

$CA_{ii} = \frac{TA_{ii}}{\rho\nabla L^2\sqrt{g/L}}$ i ∈ 4,6

- • résultats de J.H.VUGTS (modèle complet)
- o résultats de J.H.VUGTS (somme des sections)
- c résultats de M.S.CHANG
- x résultats des présents calculs
- —— Méthode des tranches

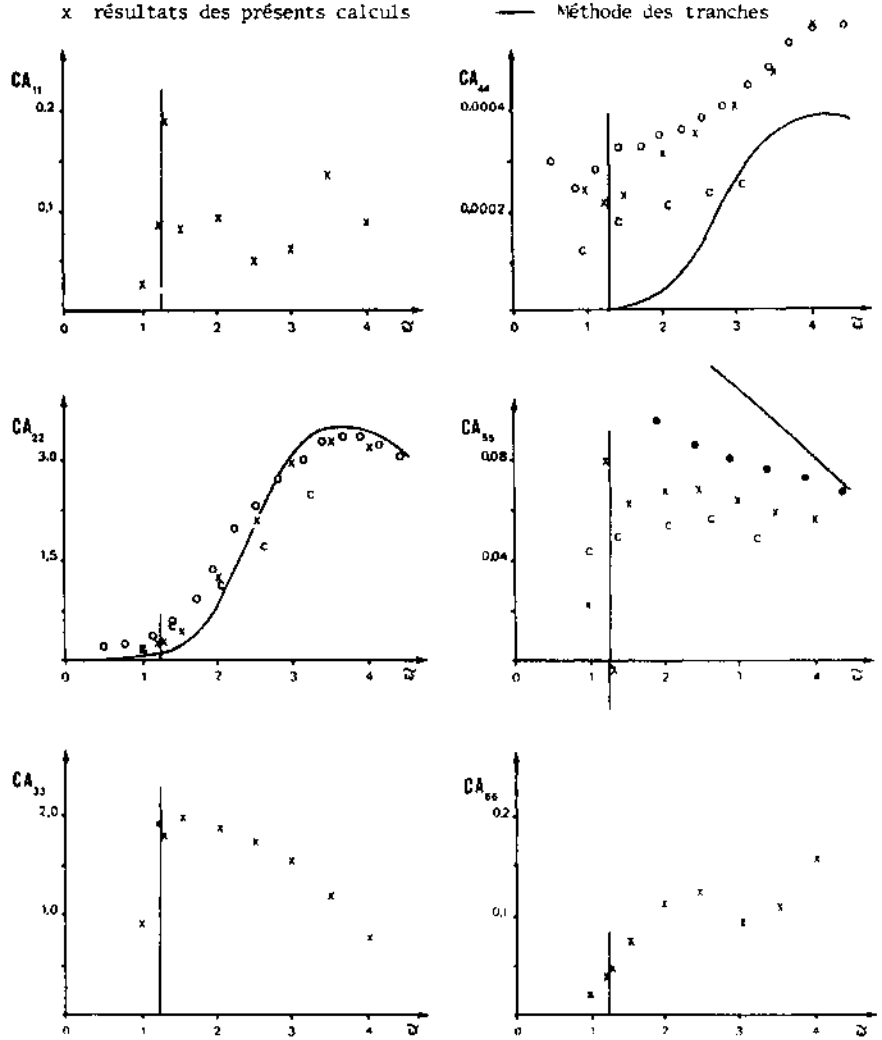


Figure 9.

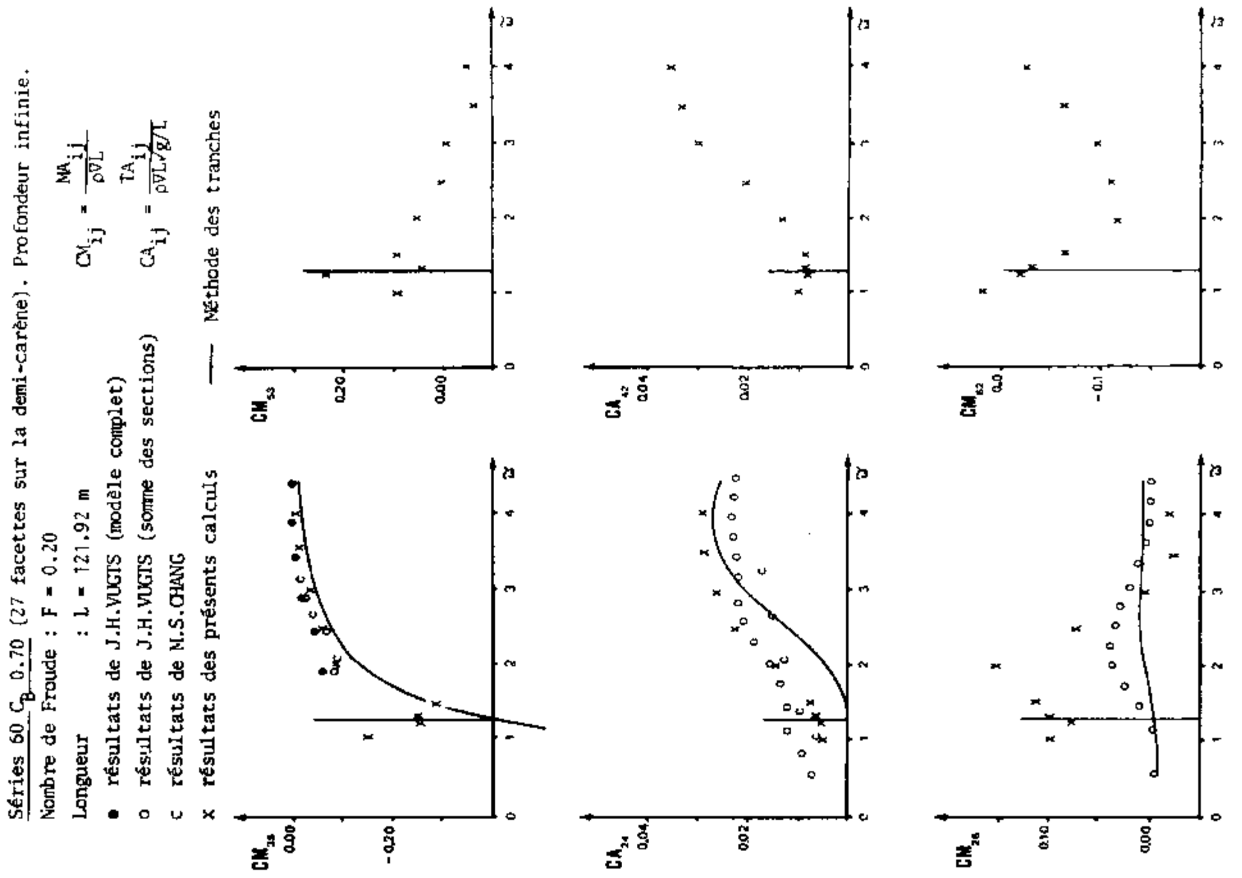


Figure 10.

14. Conclusion

The results of this work have shown that the first order motions in regular wave with forward speed can be predicted with fairly good accuracy.

Now, our computer program can be developed for calculating added wave resistance and hydroelastic response of marine structures.

An important work has been made to allow an industrial use of this computer program named DYNAPLOUS which is now working properly.

References

1. Abramowitz, M. and Stegun, I., (1967), 'Handbook of mathematical functions', Dover Publications.
2. Bai, KJ. and Yeung, R.W., (1974), 'Numerical solutions to free-surface flows problems', 10th Symposium on Naval Hydrodynamics, Cambridge Massachusets.
3. Bougis, J., (1978), 'Application de la méthode des tranches a la détermination des forces et moments de derive sur houle d'un navire au point fixe', Rapport de Recherche, Nantes.
4. Bougis, J. and Clement, A., (1979), 'Action de la houle sur un flotteur élancé à Froude zéro en profondeur finie', Bulletin de 1'ATMA, Paris.
5. Bougis, J., (1980), 'Etude de la diffraction radiation dans le cas d'un flotteur indéformable animé d'une vitesse moyenne constante et sollicité' par une houle sinusoidale de faible amplitude', These de Docteur-Ingenieur, Nantes, July 1980.
6. Brard, R., (1948), 'Introduction a 1'étude théorique du tangage en marche', Bulletin de 1'ATMA, Paris.
7. Brard, R., (1972), The representation of a given ship form by singularity distribution when the boundary condition on the free surface is linearized', Journal of Ship Research, Vol. 16, No. 1.
8. Chang, M.S. and Pien, P.C., (1975), 'Hydrodynamic forces on a body moving beneath a free surface', Conference on Computational Ship Hydrodynamics, Berkeley.
9. Chang, M.S., and Pien, P.C. (1976), 'Velocity potentials of submerged bodies near a free surface - Application to wave - Excited forces and motions', llth Symposium on Naval Hydrodynamics, London.
10. Chang, M.S., (1977), 'Computations of three-dimensional ship motions with forward speed', 2nd Conference on Computation Ship Hydrodynamics, Berkeley.
11. Courant, R. and Hilbert, D., (1937), 'Methoden der mathematischen Physik', (2 volumes) Springer, Berlin.
12. Delhommeau, G., (1978), 'Contribution a 1'étude théorique et a la résolution numérique du problème de la résistance de vagues', These de Docteur-Ingénieur, Nantes.
13. Euvrard, D., Jami, D., Morice, C. and Ousset, Y., (1977), 'Calcul numérique oscillations d'un navire engendrées par la houle', Journal de Mécanique, Vol. 16, nos. 2 et 3, Paris.
14. Guevel, P., (1979), 'Hydrodynamique navale', Cours de la Section Spéciale d'Hydrodynamique Navale Avancées, Nantes.
15. Guevel, P., Vaussy, P. and Kobus, J.M., (1974), The distribution of singularities kinematicaliy equivalent to a

moving hull in the present of a free surface', International Shipbuilding Progress, Vol. 21, No. 243.

16, Guevel, P., (1978), 'Compléments de mathématiques', Cours de la Section Spéciale d'Hydrodynamique Navale, Nantes.

17, Guevel, P. and Delhommeau, G., (1977), 'Méthodes de calcul de la résistance de vagues', Rapport I.R.C.N. Nantes.

18, Guevel, P., Daubisse, J.C. and Delhommeau, G., (1978), 'Oscillations des corps flottants soumis a 1'action de la houle', Bulletin de 1'ATMA, Paris.

19, Guevel, P., Bougis, J. and Hong, D.C., (1979), 'Formulation du problème des oscillations des corps flottants anime's d'une vitesse de route moyenne constante et sollicité' par la houle', 4th Congrès Francais de Mécanique, Nancy.

20, Havelock, T.H., (1958), The effect of speed of advance upon the damping of the heave and pitch', Transactions of the Royal Institution of Naval Architects, Vol. 100.

21, Hess, J.L., (1970), The problem of three-dimensional lifting potential flow and its solution by means of surface singularity distribution', Theoretical Aerodynamics Section, Douglas Aircraft Company.

22, Kellog, OD., (1929), 'Foundations of potential theory', Frederick Ungar Publishing Company, New York.

23, Kobus, J.M., (1976), 'Application de la méthode des singularite's au problème des flotteurs cylindriques soumis a des oscillations harmoniques forceps de faible amplitude', These de Docteur-Ingénieur, Nantes.

24. Lebreton, J.C. and Margnac, A., (1968), 'Calcul des mouvernents d'un navire ou d'une plateforme amarrés dans la houle'. La Houille Blanche, no. 5.

25. Necas, J., (1967), 'Les méthodes directes en théorie des equations elliptiques', Masson & Cie. Paris.

26. Newman, J.N., (1977), Marine hydrodynamics', The MIT Press, Cambridge Massachusets.

27. Newman, J.N., (1959), The damping and wave resistance of a pitching and heaving ship', Journal of Ship Research, Vol. 3.

28. Riesz, F. and Nagy, B.Sz., (1968), 'Leçon d'analyse fonctionnelle', Gauthier-Villars, Paris.

29. Salvesen, N., Tuck, E.O. and Faltinsen, O., (1970), 'Ship motions and sea loads', Transactions SNAME, Vol. 78.

30. St.Denis, M., Willard, J. and Pierson, J., (1953), 'On the motions of ships in confused seas', Transactions SNAME, Vol. 61.

31. Vugts, J.H., (1970), The hydrodynamic forces and ship motions in waves', Ph.D. Thesis, Delft.

32. Wehausen, J.V. and Laitone, E.V., (1960), 'Surface waves', Handbuch der Physik, Vol. 9, Springer Verlag, Berlin.

33. Wehausen, J.V., (1971), The motion of floating bodies', Annual Review of Fluid Mechanics.

34. Association de Recherches Action des Elements (1978), 'Comparion of the main numerical methods in water wave diffraction and radiation', Rapport I.F.P. 25987.